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NASA CONFERENCE ON CELESTIAL MECHANICS  
U. S. Naval Observatory, Washington, D. C.  
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NASA wishes to record its sincere appreciation to all of the speakers, the discussion chairmen and the invited participants. Lastly, appreciation is extended to Dr. J. R. Gill for the use of her notes on the discussion and the secretarial assistance of Mrs. Geraldine White.

*Nancy G. Roman*

Nancy G. Roman, Chief  
Astronomy and Solar Physics

## Preface

Recently a need for the development of space technology arising from plans by the National Aeronautics and Space Administration (NASA) for lunar and planetary exploration has given renewed impetus to and imposed stringent demands on celestial mechanics.

This conference was organized to bring together some of the scientists working in the field with representatives of NASA to explore present needs and techniques in celestial mechanics and to point up new problems in which more work should be done. It was hoped thereby that discussion would be stimulated in many areas which are now yielding somewhat to investigation but which are still difficult to handle.

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**PROCEEDINGS**

NASA CONFERENCE ON CELESTIAL MECHANICS

AGENDA

Chairman: G. M. Clemence

Thursday, January 10, 1963

MORNING SESSION

9:00 A.M. OPENING REMARKS, Chairman

9:15 A.M. I. Space Needs and Techniques, Chairman of Session,  
S. Herrick, UCLA

a. Satellites, J. W. Siry, GSFC

b. Probes, T. Hamilton; P. R. Peabody, JPL

Discussion Period, S. Herrick

BREAK (15 minutes)

11:15 A.M. II. Geodetic and Selenodetic Problems, Chairman of  
Session, J. A. O'Keefe, GSFC

a. Geodetic Problems, W. Kaula, GSFC

Brief Discussion Period (if time permits),  
J. A. O'Keefe

12:15 P.M.

LUNCH

AFTERNOON SESSION

1:30 P.M. b. The Moon's Gravitational Field, G. MacDonald, UCLA

Discussion Period, J. A. O'Keefe

BREAK (15 minutes)

3:15 P.M. III. Astronomical and Computational Problems, Chairman  
of Session, P. Herget, Cincinnati

- a. Astronomical Problems, D. Brouwer, Yale
- b. Computational Problems, M. Davis, Yale

Discussion Period, P. Herget

Friday, January 11, 1963

MORNING SESSION

9:00 A.M. IV. Mathematical Problems and Periodic Orbits,  
Chairman of Session, H. Pollard, Purdue

- a. Asymptotic Orbits, J. Kevorkian,  
California Institute of Technology

Brief Discussion (if time permits), H. Pollard

BREAK (15 minutes)

- b. Periodic Orbits and Hill Curves,  
V. G. Szebehely, General Electric

Discussion Period, H. Pollard

12:00 Noon

LUNCH

AFTERNOON SESSION

1:30 P.M. V. Relativity and Gravitation, Chairman of Session,  
G. Contopoulos, Yale and Yerkes Observatories

- a. Relativity and The Nature of Gravitation,  
R. L. Kirkwood, Rand Corporation

Discussion Period, G. Contopoulos

CLOSING REMARKS, Chairman

NASA CELESTIAL MECHANICS CONFERENCE

Morning Session, January 10, 1963

Opening Remarks

The conference participants were welcomed to the U. S. Naval Observatory by the Superintendent, Captain T. S. Baskett. The Chairman of the conference, Dr. G. M. Clemence, Director of the United States Naval Observatory, made some opening announcements in which he requested each speaker to leave or mail an abstract of his paper to NASA Headquarters. He then expressed his desire that a full discussion by the participants, be held following each paper on the subject of the paper or on a related topic.



## Ia. SATELLITE ORBIT DETERMINATION

by Joseph W. Siry

Orbit determination for scientific, applications, and manned satellites was discussed. The principal environmental factors, including gravitational and atmospheric perturbations, were described. The evolution of knowledge of these effects was discussed. The effects of mission factors upon the orbit determination problem were described. In particular, attention was devoted to the effects of orbit duration, the satellite shape and areal density, and the perigee and apogee heights of the satellite orbit. Various types of tracking data which have become available were discussed. Among these are measures of direction cosine, range, azimuth, elevation, doppler frequency, range difference, right ascension, declination, hour angle, prime vertical angle, and meridian angle. Special perturbation, general perturbation, and Encke methods were discussed, with particular reference to developments which were being employed to determine orbits. Program systems used for orbit determination were described. Applications of the orbit determination systems to Explorer, Echo, Tiros, Mercury, and other space programs were described. The use of orbit determination results to glean new geophysical information concerning the earth's gravity field and its atmosphere was discussed. Attention was devoted to some of the principal discoveries. Among these are the fact that the earth is actually pear-shaped, and the fact that its atmosphere appears to respond to certain variable solar fluxes.

## Ib. PROBES AND PLANETARY RADAR

### 1. Orbit Determination for Interplanetary Spacecraft by T. Hamilton

In constructing the orbit determination computer program for the Mariner spacecraft our first objective was to guide the spacecraft from Earth to Venus within the acceptable flyby region. Secondary objectives were to improve knowledge of the constants describing Venus and the Earth-Moon system, and to determine the astronomical unit (AU).

The computation of the spacecraft position, velocity, and the corresponding ground station observables takes into account all factors recognized to be significant including solar radiation pressure, attitude control gas jet forces, atmospheric refraction, and relativistic effects. The parameter vector which is adjusted to give the minimum sum of weighted squared residuals includes the position and velocity at epoch, the three coordinates of each tracking station, the AU, the masses of earth, moon, and the target planet, three other parameters of the earth's potential, the speed of light, and the effective "reflecting area" of the spacecraft. The solution's departure from a priori estimates is constrained by the a priori covariance matrix representing previous knowledge of the parameter vector. The data weights are computed for each point and depend on time, sample spacing, counting time, range, elevation angle, and refraction correction. The factors influencing the weights are established independently of the actual mean-squared residuals.

The two-way doppler data obtained on Mariner II by the Goldstone, California, station had an RMS residual of 0.003 M/Sec at 1 sample/minute. The effective uncorrelated noise is probably less than 0.030 M/Sec.

Further analysis of the data should provide an independent determination of the AU, a significant refinement of the mass of Venus, more precise location of two of the tracking station's coordinates ( $\pm 20$  meters), and improvement in the earth-moon and Venus ephemerides.

It is anticipated that parameters estimated from planetary radar and spacecraft radio tracking data will differ from the parameters estimated from optical data in a manner which will be helpful in isolating "hidden errors" in each data type and/or analysis assumptions or techniques. The advance in the accuracy of our knowledge of the earth and solar system made possible by such resolution will be of immense value to the space exploration program.

## Ib. PROBES AND PLANETARY RADAR

### 2. Planetary Ephemerides for Space Exploration by P. R. Peabody

Planning and successful operation of interplanetary missions depends on accurate knowledge of astronomical constants and motions of the planets. Classical theories and optical observations have defined the motions of the inner planets to about  $0''.1$  and the constants to about  $5 \times 10^{-5}$ . Recently new observing instruments, interplanetary radars, capable of resolving measurements to one part in  $10^{-9}$ , have been used, notably in radar observations of Venus and of Mariner II in its flight past Venus.

The necessity of making effective use of these measurements has raised some critical problems. I will mention four.

1. The Doppler shift is one of the two fundamental radar observables, and this requires accurate velocity data, not available from classical theory. We have successfully generated velocity data by fitting special perturbation arcs to general perturbation theories, and will continue to use this technique.
2. In any case classical theories are no longer of sufficient accuracy, and development of new position theories is critical. We lean towards the use of special perturbations because of the ease and rapidity with which they can be computed, and because they provide velocity data directly, but recognize their limitation to relatively short arcs, and feel that development of new, high accuracy and high order general perturbation theories will be necessary.
3. Since the number of radar observations are still too few to determine all the constants and motions we require, it is necessary to combine classical optical data with the radar data. We are taking three approaches.
  - a. Reducing all data, radar and optical, against new, accurate provisional theories. This project will take many years.
  - b. Using the Newcomb theories as provisional and introducing the optical data in the form of normal equations already developed and published.
  - c. Using optical data only to provide a priori values and covariance matrices for the constants and motions,

and applying radar observations under careful statistical control to obtain corrections consistent with classical knowledge.

4. Any of the three approaches above is unsettled by the inconsistency between optical and radar observations, most notable in the disagreement between the Rabe and the JPL-MIT values of the A.U. Resolution of this inconsistency would seem to be the most pressing problem confronting us.

## Ib. PROBES AND PLANETARY RADAR

### Discussion

- Hamilton: Are the radar observations inaccurate?
- Peabody: The radar distance is good to  $10^{-8}$  to  $10^{-9}$  and this is a real challenge. As for ephemerides the COSPAR Ad Hoc group is to collaborate with the IAU Paris symposium for expressing views on "astronomical constants of users." One should not use the radar observations alone without reference to tables or depending on ephemerides making use of Duncombe's improvement of Venus ephemerides. Herrick recommends a compromise. Since we cannot get optical ones reduced, let us recommend that the radar people go ahead and use these.
- Herrick: JPL does numerical integration using ephemerides and thought that this method would help to correct errors in the ephemerides (the short-term effects). We might call these "astrodynamical ephemerides." The JPL results are consistent with Rabe's results rather than MIT's.
- Hamilton: About the 2-way Doppler, comparing frequency of signal sent and received, the accuracy of the data depends on the stability of the oscillator.
- Herget: Do they observe light time if ranging? Is light-sec/AU a fundamental unit?
- Duncombe: I sympathize with JPL's need. It takes eight years to observe Venus at every point in orbit and every point in earth's orbit. The orbits will actually reflect this accuracy in less time (less than 8 years). We hope to be able to combine optical and radar methods.
- Peabody: We concur, and we feel that if we combine the two types of data, we can make a least squares solution. We plan to track Venus through the next 19 months intermittently to get these observations.
- Herrick: Less than eight years may be O.K.
- Duncombe: The transit circle observations have now been reduced up to 1956 and there appears to be no hope of speeding this up.

IIa. GEODETIC PROBLEMS RELATED TO CELESTIAL MECHANICS  
by W. M. Kaula

The zonal harmonics  $J_n$  of the earth's gravitational field have been determined from satellite orbits up to  $J_9$  with great accuracy. For the tesseral harmonics, there is still a considerable scatter of results: for example, recent solutions for  $J_{22}$  vary from 0.9 to  $1.8 \times 10^{-6}$  in amplitude and from  $10^\circ\text{W}$  to  $25^\circ\text{W}$  in direction of the major axis. However, appreciable improvement is expected with recent and forthcoming satellites such as ANNA and SYNCOM.

Computer studies for geodetic satellite orbit specifications indicate (1) the perigee height should be around 1000 km; (2) the eccentricity need not be more than 0.05; and (3) inclinations should vary from  $20^\circ$  to  $90^\circ$ , with priority for about  $60^\circ$ .

To optimize tracking station distribution, time series analysis methods are being adapted to the study of the "contamination" of determination of gravitational variations by drag model, station position, and observation error when observations are non-uniformly distributed.

In analysis of observations, aspects in which improved methods would be helpful are: data aggregation; the expression of resonant perturbations of a 24-hour orbit; the manner of expression of tesseral harmonic effects for combination of results from different orbits; the approximations to the rigorous allowance for covariance between observations at different times; and the drag model, both deterministic and statistical.

## IIa. GEODETIC PROBLEMS RELATED TO CELESTIAL MECHANICS

### Discussion

- Siry: Did you make any use of radar data?
- Kaula: There was a non-uniform distribution of observations. There are observations from four or five big dishes among them, but we do not have too many of those.
- Siry: With Syncom you get a 15 mi. range ( $10^{-9}$  accuracy); and also for POGO and EGO.
- Audience: Do you get the chordal distance? Is it possible to observe "relative velocity?"
- O'Keefe: If we know satellite height, why can't we get the size of the earth? Given  $g$ ,  $m$ ,  $A$  (?) and the differential correction is in the air, not on the ground.
- Audience: On Mariner you get the radius of the earth and you get  $A$  independent of  $gM$ .
- O'Keefe: Has any one tried to work out the Poynting-Robertson effect?
- Audience: Shapiro at MIT.
- O'Keefe: Do you expect to get the mass of the moon itself?
- Hamilton: No. We plan to do it by Ranger as soon as we get a spacecraft in operation.
- Herget: From the rotation of the earth he gets the rotation of the station?
- O'Keefe: The long-period terms...
- Herget: Hansen's theory broke down on circular orbits and critical inclinations. Vanguard was pandemonium and we could not even get a program set up.

AFTERNOON SESSION, Chairman, J. A. O'Keefe, GSFC

IIB. THE MOON'S GRAVITY FIELD  
by G. J. F. MacDonald

The two fundamental questions that have been raised with respect to the moon's interior are: (1) Is the moon differentiated and to what extent is mass concentrated toward the center? (2) Is the moon capable of supporting stress differences and are these stress differences of the same order or larger than those supported by the earth? Both of the questions can be answered by the analysis of the orbit of a close lunar satellite.

Present astronomical data set broad limits to  $J_{20}$  and  $J_{22}$  but do not limit the magnitude of the higher order terms. An assessment of available data leads to

$$\alpha = \frac{C - B}{C} = 0.000 \ 23 \pm 5$$

$$\beta = \frac{C - A}{C} = 0.000 \ 6279 \pm 15$$

$$\gamma = \frac{B - A}{C} = 0.000 \ 40 \pm 5$$

where C, B, and A are the greatest, intermediate, and least moments of inertia. The physical libration thus limits the ratio

$$\frac{J_{22}}{J_{20}} = -0.11 \pm 0.03$$

where

$$J_{20} = \frac{C - \frac{1}{2}(A + B)}{M a^2}$$

$$J_{22} = \frac{1}{4} \frac{A - B}{M a^2}$$

The ratio  $\frac{C}{M a^2}$  depends on the evaluation of the moon motion of perigee



and node and is uncertain. Relevant values are

$$\begin{aligned}\frac{C}{M a^2} &= 0.56 \pm 0.14 \text{ (motion of perigee and node)} \\ &= 0.397 \text{ (uniform moon with compression)} \\ &= 0.33 \text{ (value for earth)}\end{aligned}$$

These values combined with  $\beta$  and  $\alpha$  yield

$$J_{20} = 0.000 \ 20 \pm \ 5$$

$$J_{22} = 0.000 \ 02 \pm \ 1$$

The only approach to the higher order coefficients is through an analogy with the earth's field. The non-hydrostatic components of the earth's field are supported by internal strength. Strength scales as the product of the ratios of the density, length, and surface gravity. If the moon has density inhomogenities of the same order as the earth then

$$J_{nm} = 37 J_{\oplus nm}$$

This hypothesis can be tested by comparing the calculated  $J_{20}$  and  $J_{22}$  and the observed values

	<u>CALCULATED</u>	<u>OBSERVED</u>
$J_{20}$	$4.3 \times 10^{-4}$	$2.0 (\pm 0.5) \times 10^{-4}$
$J_{22}$	$5.9 \times 10^{-5}$	$2.0 (\pm 1.0) \times 10^{-5}$

The external field of a moon having a strength comparable with the earth will have J's that are larger than those of the earth. Kaula has shown that these J's will lead to a large number of observable perturbations of a lunar orbiter.

## IIB. THE MOON'S GRAVITY FIELD

### Discussion

- Hertz: What is the period of lunar satellites?
- Danby: Is there a plan for using a transit instrument on the moon?
- MacDonald: O'Keefe believes that the moon will be "smooth." MacDonald believes that the moon will be "rough."
- Danby: Orbits around the moon should help settle this...
- Hertz: Can you get the orbits accurately enough?
- Kaula: It is harder to pick up long-period disturbances.
- Deutsch: There is an analogy with a double star orbit. The moon has a finite parallax!
- Roman: Does the moon present problems that are not present for earth satellites?
- O'Keefe: Tidal distortions.
- Roman: Additional distortions due to the figure of the moon?
- O'Keefe: Bodily tidal...
- Herget: Answer Roman's questions... (comparable effects).
- Contopoulos: Are there any calculations for orbits near the surface of the moon?
- MacDonald: 100 km is the lowest.
- Herrick: Would you define your lunar J's?
- Vinti: Ellipsoidal coordinates...
- Herrick: This brings up the IAU (International Astronomical Union) normalization discussion.

MacDonald: Isostasy.

O'Keefe: You think isostasy does exist on the moon? In the maria?

MacDonald: There is a strong zone at 500 km.

O'Keefe: Is it a liquid or not?

Audience: Is there any uniqueness theorem that gives you the shape?

MacDonald: Not without additional constraints.

Audience: The weak point in the mantle probably is the point where the moon is liquid.

O'Keefe: What caused distortions in the inner load of the mantle?

Szebehely: Uniqueness.

Contopoulos: But there are all kinds of orbits.

### IIIa. ASTRONOMICAL PROBLEMS

by D. Brouwer

1. The availability of high-speed computers presents opportunities for applications to general planetary theories. For literal developments tabulations of the Laplacian coefficients and their derivatives are required, the latter up to the order equal to the highest power in the eccentricities and inclinations to be retained in the series. Next, Newcomb's operators to the same order are needed. It may be hoped that a project undertaken at the Smithsonian Institution by Dr. Izsak and a project undertaken jointly by the U. S. Naval Observatory and the Yale Observatory may be coordinated and before long produce the desired tabulations.

2. The methods of perturbation theory are not well adapted to problems concerned with trajectories that pass close to one or more attracting bodies. Perhaps regularization will hold the answer.

3. One of the unsolved problems of celestial mechanics is that of dealing in a general manner with the solution of differential equations in which two or more small divisors occur simultaneously. The three-dimensional restricted problem in the vicinity of a commensurability  $(p+q)/p$  is such a problem.

The reduction of this problem to one or two degrees of freedom by the elimination of the periodic terms is sufficient to demonstrate that the Kirkwood gaps in the asteroid belt are an expected feature. The attack on the problem of dealing with two or more critical arguments simultaneously may show that a close relationship exists between the general distribution of asteroids according to mean motion and the distribution of principal commensurabilities.

### IIIb. COMPUTATIONAL PROBLEMS IN CELESTIAL MECHANICS

by M. S. Davis

A brief review is given of a number of computational topics that have held the attention of workers in space science for the last ten years. Mention is made of recent studies comparing numerical integrations of orbits according to Cowell's method, Encke's method and the Variation of Parameters. It is pointed out that comparisons of methods must always be made cautiously.

The accumulation of round-off errors is then discussed. With the use of high-speed computers, the particular programming language used (or the method of rounding) has an effect on the accumulation of round-off errors. Thus, in FORTRAN all floating-arithmetic operations in single precision are truncated to eight figures, which in some celestial mechanical investigations results in systematic errors. The conclusion is reached that the main numerical analysis problems in celestial mechanics are well in hand.

Great efforts in the future should be devoted towards the development of literal programs capable of solving general planetary and satellite theories. Some pioneering work has already begun and with the advent of the latest generation of computers, the solution of this problem now seems within reach.

Finally, some of the pros and cons are sketched concerning a central library of celestial mechanical programs or at least a list of titles of such programs and their whereabouts.

### IIIb. COMPUTATIONAL PROBLEMS IN CELESTIAL MECHANICS

#### Discussion

- Clemence: These machine methods are powerful, but inefficient.
- Herrick: Do you think we should use Cowell's method for total acceleration--perhaps Jackson-Gauss or the "second sum formula"? The Runge-Kutta compared to the Jackson-Gauss is not quite so favorable. There is no comparison as general as some authors think there is! Use Cowell when the perturbations change rapidly as with high e's. For instance, the orbit of Icarus by Encke's method and lunar trajectories...interplanetary orbits.
- Cohen: I am disappointed in results of perturbation problems from the Cowell Method. There is no improvement by going to Variation of Parameters.
- Herrick: Cowell's method is often the best.
- Herget: I would put the reason a bit differently.
- Hertz: Is there any drag in the two-body problem?
- Davis: In single precision.
- Hertz: Using ALGOL instead of FORTRAN?
- Herget: Astronomers have been doing the right thing all along. Gauss showed us!
- Oesterwinter: We have 7090 programs...
- Contopolous: How could these formulae (H - K) be generalized... for the case of Jupiter?

MORNING SESSION, Chairman, H. Pollard, Purdue

IVa. EARTH-MOON TRAJECTORIES IN THE RESTRICTED THREE-BODY PROBLEM  
by J. Kevorkian

Planar motion of a particle of negligible mass from the neighborhood of a gravitational center (the "earth") of mass  $(1-\mu)$  to the neighborhood of a second center (the "moon") of mass  $\mu$  is studied within the framework of the restricted three-body problem by asymptotic methods for the case  $\mu \ll 1$ .

It is shown that there exist two regions centered around the earth and moon respectively, and two associated approximations of the exact equations which to order unity are Keplerian relative to their corresponding centers.

It is pointed out that in order to determine the motion near the moon and hence the subsequent motion it is necessary to compute the trajectory leaving the earth correct to order  $\mu$  (i.e. the Keplerian conic relative to the earth plus a correction to order  $\mu$ ). This is due to the dependence of the angular momentum for the hyperbolic orbit to order unity around the moon upon quantities of order  $\mu$ .

It is shown that the two asymptotic developments thus obtained match directly after one has computed the behavior of the expansion relative to the earth near the moon. This might require the numerical evaluation of certain functions given in integral form.

Having derived the two pertinent expansions near each of the attractive centers it is a straightforward matter to write down the composite expansion for the entire trajectory which will be uniformly valid throughout space.

#### IVa. EARTH-MOON TRAJECTORIES IN THE RESTRICTED THREE-BODY PROBLEM

##### Discussion

Contopolous: How fundamental is this method?

Kevorkian: You can't tell what happens. You get periodic orbits with small  $\mu$ 's. It is capable of going to higher order terms, but it will be hard to do this because you will have the same difficulty. The intermediate solution is contained in the outer solution and this is the crux of the matter.

Herrick: I wish to congratulate the speaker on not using rotating systems, and to ask him what relation there is between the method of variation of parameters and the substitution of approximate solutions in here?



## PERIODIC SOLUTIONS

by R. Arenstorf

One can get mathematically exact solutions in rotating coordinate systems and neglect the mass of the earth. This can be done to predict periodic solutions (ellipses) around earth and moon using the Poincare method with periodic orbits of order  $\mu$  (small mass). Proofs are now available for Poincare orbits of the second kind with small  $e$ . "Small" means between  $\mu^*$  and 0.  $\mu^*$  is still not defined, but the range is beginning to be understood. Families (of orbits) do not just suddenly disappear. (Six-decimal accuracy is being used). A complete proof (existence theorem) will be found in the Journal of Mathematics, and graphs are being published in the AIS Journal.

\* By request of several participants Arenstorf gave an informal discussion of work he has been doing on periodic orbits.

IVb. PERIODIC ORBITS AND HILL-CURVES  
by V. G. Szebehely

Relations between zero velocity curves and orbits are investigated. A general condition which the force function (or the potential) must satisfy is derived. This condition is in general expressed as a second order partial differential equation of fourth degree in the force function of the particular problem.

The zero velocity curves for the restricted problem of three bodies are called Hill-curves, and the question is discussed; under what conditions will these curves become orbits? If the Hill-curves, which are also orbits, are closed, then periodic orbits are generated.

Several special cases are discussed. Regarding conservative, two degrees of freedom dynamical systems the general formulation of the above-mentioned condition is given and one special result is mentioned. This is the field with potential  $V = Cr^n \exp(ik\theta)$ , where the equipotential lines and the orbits are logarithmic spirals. Here  $C$ ,  $k$ , and  $n$  are constants,  $r$  is the radial and  $\theta$  is the angular polar coordinate.

Regarding the restricted problem, the following three results are derived. 1. Considering the case of "small" mass ratio ( $\mu$ ), expanding the force function by means of Legendre polynomials, and applying the derived condition when terms of  $\theta(\mu^2)$  are neglected, shows that for motion far from both primaries the Hill-curves are orbits if terms of  $\theta(r^{-3})$  are omitted. 2. For motion around either of the primaries, terms of  $\theta(r^2)$  are to be omitted in the Legendre expansion in order to obtain agreement between Hill-curves and periodic orbits. 3. Regarding periodic orbits around the triangular libration points, it can be shown that, for the linearized case, the Hill-curves are ellipses and so are the orbits. The eccentricities and the orientations of the axes of these two sets of ellipses, however, are not the same. Therefore, the Hill-curves are not periodic orbits around the triangular libration points in the linear treatment.

AFTERNOON SESSION, Chairman, G. Contopoulos, Yale and Yerkes Observatories

V. RELATIVITY AND THE NATURE OF GRAVITATION

by R. L. Kirkwood

Starting with classical ideas of space, time, and gravity, Einstein's principle of equivalence is shown to lead to a description of gravity in terms of an ether flow. This description, when combined with some of the well-established results of the special theory of relativity, is shown to lead to all of the verifiable results of the general theory of relativity. It also leads to a formulation of mechanics in which a body in a gravitational field moves along one of the four-dimensional geodesics associated with the time element measured by a physical clock. Similarly, the path of a ray of light is a zero-length geodesic associated with the same time element. Finally, it is shown that when gravity is described by an ether flow plus a scalar potential function, the result is equivalent to Einstein's theory if the three-dimensional geometry is Euclidean. Thus Einstein's theory is interpreted as an ether flow in a three-dimensional Riemannian space, although the additional complexity of the three-dimensional Riemannian geometry appears to be quite unnecessary.

## V. RELATIVITY AND THE NATURE OF GRAVITATION

### Discussion

Contopolous: This is of general interest... there have been many attempts to verify relativity theory. This is different from Einstein's approach...what are the advantages?

Kirkwood: It is a satisfying physical picture...it gives and suggests a different line of approach. Einstein's approach leads to Riemannian geometry and I feel this is unfortunate because it is so far from physical facts.

Contopoulos: Not covariant?

Kirkwood: Basically, but what does it mean?

Dicke: A difference between out-flowing or in-flowing ether? Have you looked at the precession of the gyroscope, that is, the field?

Clemence: If precession is measured and it agrees with Einstein, would you give up this approach?

Kirkwood: No.

NASA CONFERENCE ON CELESTIAL MECHANICS  
January 10 and 11, 1963

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W. M. Kaula, Goddard Space Flight Center  
J. Kevorkian, California Institute of Technology  
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R. H. Lyddane, Naval Weapons Laboratory  
G. MacDonald, University of California, Los Angeles  
B. Marsden, Yale University  
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